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Effective media formation and conduction through unsaturated granular materials

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Abstract—A theory for the effective thermal conductivity (ETC) of an unsaturated and saturated medium containing three phases is presented using the effective continuous media (ECM) approximation. The boundary condition for ECM through a successive dispersion technique is first derived. Small dispersions are then allowed in the proposed ECM. The method is an analytical one and shows the validity and emergence of ECM for large dispersions. The theory is applicable for all saturating fluids including gas and is useful for determining the content of petroleum products in underground soil. A comparison of calculated values of ETC with experimental ones when the saturating fluids are air and water shows a good agreement.

INTRODUCTION

THE EFFECTIVE properties of a heterogeneous two-phase medium have been explored in the literature [1–10] using generalized functions [1, 2], multipole expansions [3, 4] and volume averaging approaches [5–8]. In all these methods, except in refs. [9, 10], authors considered dispersion of another phase in a continuous phase. This kind of approach is valid only till the continuous medium has a majority character. At large dispersions like loose and granular materials, metal shots, moist soil, etc. the majority character loses its meaning. A similar behaviour may be observed in the case of saturated porous materials. Therefore, the theories need a modification. The integrated theory [11–13] using lattice type dispersions, derived particularly for loose and granular materials, is therefore extended to serve the need for an unsaturated two-phase medium.

The natural two-phase saturated and unsaturated granular materials are mixtures of solid and fluid phases where each of the phases occupy a large volume fraction (0.3–0.7) of the sample. In this situation none of the phases (solid or liquid) provide the continuous medium. The continuous medium that persists at this porosity is the effective continuous medium (ECM) composed by both the phases. The effective continuous media may thus be defined as an ordered homogeneous medium composed by equal volume fractions of both phases. By allowing a small dispersion of the solid or liquid phase in ECM one thus enables the actual two-phase system to be generated.

An unsaturated media is a three-phase system composed by solid, liquid and gas. It results in a two-phase media when it is saturated by either phase. The process is equivalent to the variable dispersion of the liquid phase in a dry granular powder. The difficulty confronted in the experimental determination of effective thermal conductivity (ETC) of unsaturated medium is the intertwining of fluid convection and conduction in a complicated way. The flow path of conduction and convection is not the same. The situation becomes more complicated due to adsorption, capillarity and surface tension.

A complete microscopic description of the unsaturated media taking into account all these processes is a very difficult task. The procedure adopted here is an analytical one and is supported by the outcome of ETC measurements. At low liquid contents there occurs adsorption of liquid around the solid surface. Distribution of fluid around the solid grain is uniform and is proportional to the specific area of the solid grain. However, at high fluid content there is a fluid bridging between two solid grains. This is equivalent to the short circuiting of the resistances. It enhances the ETC of the system very rapidly. Thus an unsaturated medium is to be looked at from different angles at low and high liquid contents.

THEORETICAL ANALYSIS

The procedure adopted here is to derive first the ETC of the fluid saturated granular medium. The small dispersions of the liquid or gas phase are then allowed to occur in the fluid saturated medium. Small dispersions of the liquid phase in a gas saturated granular medium yield a low fluid unsaturated medium. Similarly small dispersions of the gas phase in a liquid saturated medium yield a high fluid unsaturated medium.

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NOMENCLATURE

- a ratio of thermal conductivity values of solid with respect to fluid
- b coefficient of ECM
- $\langle E \rangle$ volume averaged field in a dispersed medium
- E_0 volume averaged field in a single phase medium
- $\langle \mathrm{d}E \rangle$ change in volume averaged field due to dispersion
- n total number of successive dispersions
- r_i distance between two solid spheres in lattice model
- S_r degree of saturation.

Greek symbols

- λ_a thermal conductivity of air
- λ_c thermal conductivity of a single phase continuous medium
- λ_d thermal conductivity of a dispersed medium
- λ_f thermal conductivity of fluid
- λ_g thermal conductivity of gas
- λ_l thermal conductivity of liquid
- λ_{ef} effective thermal conductivity of a dispersed medium
- λ_{st} thermal conductivity of a saturated two-phase medium
- λ_s thermal conductivity of solid phase
- λ_w thermal conductivity of water
- λ_{ecm} thermal conductivity of ECM
- λ_{ecmg} thermal conductivity of ECM formed by a gas–solid system
- λ_{ecmw} thermal conductivity of ECM formed by a water–solid system
- λ_{ecmf} thermal conductivity of ECM formed by a fluid–solid system
- λ_{gl} thermal conductivity of a gas–liquid system
- ξ amount of volume fraction which is higher than 0.5
- $\delta\psi_s$, $\delta\psi_g$, $\delta\psi$ small dispersions
- ψ_s volume fraction of solid phase
- ψ_g or ψ_l volume fractions of gas or liquid phases
- ψ_d volume fraction of a dispersed phase.

ETC of fluid–solid system

In this section one finds the ETC of a two-phase fluid–solid system using the same steady and constant potential source kept first in a single phase system (λ_c) and then in a two-phase fluid–solid system using a lattice model [12]. Here the dispersion of the second phase in a continuous phase is quite small and it does not affect the majority character of the continuous phase. In that case, let $\bar{E}_0$ be the field near the source in the single phase system and $\langle \bar{E} \rangle$ be the field in the two-phase system, one finds that in the steady state

$$\begin{aligned} \langle \bar{J} \rangle &= \lambda_c \bar{E}_0 \\ &= \lambda_{ef} \langle \bar{E} \rangle \end{aligned} \tag{1}$$

and if $\langle \bar{E} \rangle = \bar{E}_0 \mp \langle \mathrm{d}\bar{E} \rangle$ one may write

$$\lambda_{ef} = \frac{\lambda_c}{1 \pm \langle \mathrm{d}\bar{E} \rangle / \bar{E}_0} \tag{2}$$

where negative and positive signs stand for the cases $\lambda_d > \lambda_c$ and $\lambda_d < \lambda_c$, respectively [13]. Applying perturbation treatment to equation (2) one finds that for $\langle \mathrm{d}\bar{E} \rangle / \bar{E}_0$ less than unity

$$\lambda_{ef} = \lambda_c (1 \pm \langle \mathrm{d}\bar{E} \rangle / \bar{E}_0). \tag{3}$$

For lattice type dispersions, where one considers a dispersion of regular and finite spaced spheres in cubic arrays, the contribution to $\langle \mathrm{d}\bar{E} \rangle$ occurs only due to spherical particles situated in the plane of the observation point as the flow of energy along the source of constant potential is zero. As the thermal conductivity values of the considered fluids in general (water, air) are very small as compared to the solid phase (metal shots, glass, soil) the contribution to $\langle \mathrm{d}\bar{E} \rangle / \bar{E}_0$ is estimated using Green's function technique [12] directly from the geometry. This yields

$$\langle \mathrm{d}\bar{E} \rangle / \bar{E}_0 = \sum_{i=1}^{4} (3/4\pi) \frac{\lambda_d - \lambda_c}{\lambda_d + 2\lambda_c} \int \bar{\nabla}(1/r_i)\, \mathrm{d}\bar{S}. \tag{4}$$

As the field modification $\langle \mathrm{d}\bar{E} \rangle$ in the case of dilute dispersion is limited ($\lambda_f / \lambda_s \to 0$) to the overlapping contribution from a few nearest neighbours only, one finds the ETC of two-phase media by summing equation (4) up to the fourth nearest neighbours

$$\lambda_{ef} = \lambda_c \{1 + 3.8440(\lambda_d - \lambda_c / \lambda_d + 2\lambda_c) \psi_d^{2/3}\}. \tag{5}$$

ECM approximation

Equation (5) is valid for small values of ψ_d such that λ_c still represents a majority medium. Moreover, fluid saturated granular systems have a very high porosity (0.3–0.7). The majority character of the continuous phase at such large dispersions is totally lost. It is considered that there is an ordered and homogeneous medium called ECM which is composed of equal volume fractions of solid and liquid phases. By allowing a small dispersion of the solid or fluid phase in ECM (where $\psi_s = \psi_f = 0.5$) one may generate the actual saturated two-phase system.

ETC of ECM and successive dispersion

If n successive small dispersions of solid (λ_s) or fluid phases (λ_f) are allowed to occur in either phase till $n\delta\psi_s$ or $n\delta\psi_f$ is 0.5 the resulting medium is ECM. Under this condition equation (5) remains valid. The thermal conductivity of the resulting ECM in two different cases following equation (5) would be

$$\lambda = \lambda_s\{1+3.844(\lambda_f-\lambda_s)/(\lambda_f+2\lambda_s)\delta\psi_f^{2/3}\}^n = \lambda_f\{1+3.844(\lambda_s-\lambda_f)/(\lambda_s+2\lambda_f)\delta\psi_s^{2/3}\}^n. \quad (6)$$

As a fluid is less interacting as compared to a solid the coefficient 3.844 in the case of fluid dispersion in a solid reduces to 3.09 [12]. Moreover, the harmonic and arithmetic means of the two values of equation (6) only account for the contribution of the solid phase, hence the geometric mean is the only useful value. For $\delta\psi_f = \delta\psi_s = \delta\psi$ the geometric mean yields

$$\lambda^2 = \lambda_s\lambda_f\left\{1+3.844\left(\frac{\lambda_s-\lambda_f}{\lambda_f+2\lambda_s}\right)^2\frac{1}{(\lambda_s+2\lambda_f)}\delta\psi^{2/3} - 3.844\frac{(\lambda_s-\lambda_f)^2}{(\lambda_s+2\lambda_f)}\frac{1}{(\lambda_f+2\lambda_s)}(\delta\psi^{2/3})^2\right\}^n. \quad (7)$$

When the saturating fluid is a gas $\lambda_f/\lambda_s = \lambda_g/\lambda_s \to 0$ (for increasing powers of λ_g/λ_s) and equation (7) reduces to [11]

$$\lambda^2 = \lambda_s\lambda_f\{1+2.299(n\delta\psi^{2/3})-5.939(n\delta\psi^{2/3})^2/n\}. \quad (8)$$

In case the saturating fluid is a liquid equation (7) transforms into

$$\lambda^2 = \lambda_s\lambda_l\left[\left\{1+1.922\left(\frac{a-2}{a+2}\right)n\delta\psi^{2/3} + 1/2\left\{1.922\left(\frac{a-2}{a+2}\right)\right\}^2(n\delta\psi^{2/3})^2 - 7.388\left(\frac{a-2}{a+2}\right)(n\delta\psi^{2/3})^2/n\right\}\right] \quad (9)$$

where

$$\lambda_s/\lambda_f = a$$

and

$$\{(a-1)^2/(a+2)(2a+1)\} \to a(a-2)/2a(a+2)$$

for $a \gg 1$.

Evaluating $n\delta\psi^{2/3}$ for $n\delta\psi = 0.5$ and $\delta\psi > 0$ [11] one finds the ETC of ECM of the gas–solid two-phase system through equation (8) as

$$\lambda_{ecmg} \geqslant 0.6132(\lambda_s\lambda_g)^{1/2}. \quad (10)$$

Similarly for the same values of $n\delta\psi^{2/3}$ equation (9) yields the ETC of ECM of the fluid–solid system as

$$\lambda_{ecmf} \geqslant (\lambda_s\lambda_f)^{1/2}\left[1-1.41183\left(\frac{a-2}{a+2}\right)+0.848250\left(\frac{a-2}{a+2}\right)^2\right]^{1/2}. \quad (11)$$

For $\lambda_f = \lambda_w = 0.6$ and $\lambda_s = 4.16$ (soil) equation (11) reduces to

$$\lambda_{ecmw} \geqslant 0.6924(\lambda_s\lambda_w)^{1/2}. \quad (12)$$

One notices through equations (10) and (12) that as the value of λ_f approaches closer to λ_s, the value of the constant in equation (10) approaches unity. Thus in general

$$\lambda_{ecmf} \simeq b(\lambda_s\lambda_f)^{1/2} \quad \text{where} \quad 0.6132 \leqslant b \leqslant 1. \quad (13)$$

ETC of fluid saturated two-phase system

When $\psi_f > 0.5$ in the saturated medium, a small dispersion of the fluid phase $\xi_f = \psi_f - 0.5$ is allowed to occur in ECM. It produces the desired saturated medium. The thermal conductivity of this medium is given using equation (5) as

$$\lambda_{st} = b(\lambda_s\lambda_f)^{1/2}\left\{1+3.844\left(\frac{\lambda_f-\lambda_{ecmf}}{\lambda_f+2\lambda_{ecmf}}\right)\xi_f^{2/3}\right\}. \quad (14)$$

For gas and liquid $b = 0.6132$ and 0.6924, respectively.

However, if $\psi_s > 0.5$ in saturated media, a small volume fraction of solid $\xi_s = \psi_s - 0.5$ is allowed to disperse in ECM to produce the desired saturated two-phase medium. Following equation (5) the thermal conductivity is given as

$$\lambda_{st} = b(\lambda_s\lambda_f)^{1/2}\left\{1+3.844\left(\frac{\lambda_s-\lambda_{ecmf}}{\lambda_s+2\lambda_{ecmf}}\right)\xi_s^{2/3}\right\}. \quad (15)$$

Unsaturated medium with variable fluid contents

(a) *Low liquid contents.* Due to adsorption, liquid is distributed evenly around the solid surface in this case. It replaces the gas in the void space and changes the thermal conductivity of the void space. Let ψ_{gl} be the volume fraction of liquid with respect to the total void (ψ_g) in the sample. The thermal conductivity (λ_{gl}) of the void which is the gas–fluid system is given by equation (5)

$$\lambda_{gl} = \lambda_g\left\{1+3.844\left(\frac{\lambda_l-\lambda_g}{\lambda_l+2\lambda_g}\right)\psi_{gl}^{2/3}\right\} \quad (16)$$

where $\psi_{gl} = \psi_l/\psi_g = S_r$ (degree of saturation) and $(\lambda_l-\lambda_g)/(\lambda_l+2\lambda_g) \to 1$.

The thermal conductivity of ECM also changes, as it strongly depends upon the thermal conductivity of the void (equations (10), (12) and (13)). Following equation (10) λ_{ecm} now becomes

$$\lambda_{ecm} = 0.6132(\lambda_s\lambda_{gl})^{1/2}. \quad (17)$$

Table 1. Thermal conductivity of unsaturated and saturated soils ($W\ m^{-1}\ K^{-1}$) [22]

Soil type	System	λ_s	S_r	ψ_a	λ_{cal} Kersten [14]	λ_{cal} Johansen and Frivick [16]	λ_{cal} deVries [9]	λ_{cal} Present theory	λ Experimental
1. Unsaturated frozen (three phases) without unfrozen water	Ramsey Sandy loam [14]	4.15	0.083	0.476	0.282	0.432	0.965	0.333	0.247
			0.119	0.399	0.445	0.601	1.311	0.468	0.519
			0.147	0.350	0.602	0.740	1.567	0.542	0.679
			0.343	0.346	0.940	1.350	2.295	1.193	1.138
			0.489	0.289	1.343	1.893	2.814	1.692	1.507
			0.686	0.286	1.654	2.515	3.123	2.012	1.507
			0.744	0.340	1.618	2.596	3.045	1.920	1.803
			0.858	0.246	2.056	3.133	3.451	2.464	2.513
			0.972	0.276	2.113	3.429	3.515	2.669	2.493
			1.000	0.346	2.034	3.372	3.411	2.663	2.391
2. Unsaturated frozen (three phases) without unfrozen water	Fairbanks Silty Clay loam [14]	5.0	0.037	0.659	0.112	0.210	0.480	0.136	0.140
			0.063	0.528	0.210	0.359	0.855	0.236	0.235
			0.172	0.531	0.430	0.699	1.317	0.272	0.420
			0.225	0.475	0.564	0.922	1.694	0.432	0.522
			0.327	0.527	0.743	1.187	1.803	0.365	0.632
			0.411	0.467	0.925	1.534	2.260	0.754	0.870
			0.541	0.404	1.214	2.078	2.829	1.194	1.201
			0.595	0.469	1.277	2.131	2.687	1.597	1.276
			0.798	0.397	1.681	2.964	3.354	1.723	1.696
			0.841	0.470	1.750	2.931	3.195	1.638	1.596
			0.928	0.440	1.907	3.289	3.459	2.105	1.849
			1.000	0.467	2.053	3.455	3.529	2.354	1.908

3. Unsaturated frozen	Penner	3.0	0.174	0.486	0.454	0.610	1.056	0.306	0.361
(three phases)	*et al.*		0.209	0.470	0.533	0.706	1.157	0.354	0.511
without unfrozen	[15]		0.372	0.269	1.290	1.290	2.058	1.218	1.168
water			0.442	0.441	0.995	1.298	1.741	0.627	0.982
			0.513	0.368	1.203	1.528	2.030	1.046	1.815
			0.596	0.420	1.293	1.690	2.057	0.990	0.948
			0.778	0.421	1.625	2.135	2.343	1.325	1.660
			0.806	0.394	1.688	2.224	2.422	1.479	1.591
			0.900	0.397	1.854	2.451	2.555	1.614	1.901
			1.000	0.281	2.184	2.781	2.787	2.250	2.581
4. Unsaturated unfrozen	Kersten	4.55	0.051	0.501	0.599	0.314	0.660	0.260	0.245
(three phases)	Dakota		0.082	0.410	0.900	0.648	0.937	0.456	0.460
	Sandy loam		0.096	0.349	1.083	0.844	1.139	0.519	0.554
	[22]		0.138	0.291	1.426	1.213	1.443	1.040	0.883
			0.240	0.289	1.817	1.575	1.678	1.135	1.289
			0.288	0.274	2.007	1.751	1.832	1.218	1.318
			0.325	0.290	2.023	1.765	1.911	1.190	1.478
			0.468	0.221	2.648	2.325	2.486	1.555	2.271
			0.590	0.290	2.439	2.148	2.221	1.393	2.164
			0.727	0.190	3.244	2.821	2.910	1.837	2.722
			0.825	0.234	3.073	2.659	2.718	1.218	2.640
			0.886	0.294	2.696	2.392	2.452	1.676	2.363
5. Saturated unfrozen	Penner	1.70	1.000	0.571	1.060	0.903	0.941	0.775	0.800
(two phases)	Leda			0.482	1.292	0.997	1.036	0.916	0.920
	Clay			0.498	1.337	0.980	1.019	0.851	0.930
	[22]			0.447	1.464	1.037	1.076	1.010	0.991
6. Saturated frozen	Johansen	4.00	1.000	0.338	3.351	3.321	3.353	2.855	3.111
(two phases)	Sand			0.338	3.351	3.480	3.510	2.840	2.678
	SA_2								
	[22]								

The ETC of an unsaturated medium is given by making a small dispersion $\xi_s = \psi_s - 0.5$ or $\xi_g = \psi_g - 0.5$ of solid or gas phases, respectively, for $\psi_s > 0.5$ or $\psi_g > 0.5$ as the case may be. Following equation (5) one finds

$$\lambda_{ef} = \lambda_{ecm}\left\{1+3.844\left(\frac{\lambda_s-\lambda_{ecm}}{\lambda_s+2\lambda_{ecm}}\right)\xi_s^{2/3}\right\}$$

for $\psi_s > 0.5$

$$\lambda_{ef} = \lambda_{ecm}\left\{1+3.844\left(\frac{\lambda_g-\lambda_{ecm}}{\lambda_g+2\lambda_{ecm}}\right)\xi_g^{2/3}\right\} \quad (18)$$

for $\psi_g > 0.5$

$$\lambda_{ecm} = 0.6924(\lambda_s\lambda_l)^{1/2}.$$

Using equations (16)–(18) one finds the ETC of unsaturated three-phase media.

(b) *High liquid content*. At high liquid contents there is short circuiting due to liquid bridging between two solid grains. It reduces the resistivity of the medium very quickly. The liquid surrounds the solid grains and instead of affecting the ETC of the void it starts to form an ECM of the liquid–solid system. Gradually as the amount of liquid is increased the amount of void gas decreases. At complete liquid saturation the void is completely free from gas. Thus the system is like gas dispersion in liquid saturated media. The thermal conductivity of ECM formed by a liquid–solid system is given using equation (10) as

$$\lambda_{ecm} = 0.6924(\lambda_s\lambda_l)^{1/2}.$$

If $\psi_s > 0.5$ the unsaturated media is formed by making $\xi_s = \psi_s - 0.5$ the dispersion of the solid phase in ECM. Using equation (15) one finds the thermal conductivity of unsaturated media (λ_{ef}) as

$$\lambda_{ef} = 0.6924(\lambda_s\lambda_l)^{1/2}\left\{1+3.844\left(\frac{\lambda_s-\lambda_l}{\lambda_s+2\lambda_l}\right)\xi_s^{2/3}\right\}. \quad (19)$$

However, if an unsaturated medium is formed by making a gas dispersion of $\xi_g = \psi_g - 0.5$ then equation (19) becomes ($\lambda_a = \lambda_g$)

$$\lambda_{ef} = 0.6924(\lambda_s\lambda_l)^{1/2}\left\{1+3.844\left(\frac{\lambda_a-\lambda_l}{\lambda_a+2\lambda_l}\right)\xi_g^{2/3}\right\}. \quad (20)$$

COMPARISON AND RESULTS

The estimated values of ETC using the equations derived in the present analysis are compared with the experimental data on air, water and ice saturated systems. In the case of unsaturated media a number of results are available [14–16] for a variety of soils. As there are also some other models for soils [9, 14, 16–18] a specific comparison of the ETC of different soils using these models too are made in Table 1.

Using the present model all calculations have been carried out with $\lambda_a = 0.026$ W m^{-1} K^{-1}, $\lambda_w = 0.6$ W m^{-1} K^{-1} and $\lambda_{ice} = 2.2$ W m^{-1} K^{-1} while the data of refs. [9, 14, 16] reported as a comparison have been obtained by Farouki [22] with $\lambda_a = 0.046$ W m^{-1} K^{-1} for soil types 1, 2 and 4 and $\lambda_a = 0.024$ W m^{-1} K^{-1} for soil types 3, 5 and 6.

The ETC values for an air saturated medium are compared in Table 2. It is worth noting that the percentage error between theoretical and experimental results using the present model lie between 0 and 10.9%. One also notes the incapability of the theory of dilute dispersion [8] and single phase continuous media for the estimation of ETC of air saturated materials.

In Table 1 ETC values of unsaturated and saturated soils are compared. For Ramsey Sandy loam [14] unsaturated frozen soil, except at low water content, the estimated values using the present theory are always in better agreement as compared to the other theories. The maximum error leaving data 1 and 6, in this case is 20.2%. In the case of Fairbanks Silty Clay loam [14] the results are much better than the previous ones at low water content. The maximum error, except data 3 and 5, in this case is 25.2%. Here the estimated values by Kersten [14] are also in agreement with the experimental results but at low water content the present theory seems more acceptable. In the case of Penner *et al.* [15], unsaturated frozen soil, the estimated values using the present theory are in agreement with experimental results except data 2, 4 and 5. The maximum error in this case is about 20.2%. Also, in this case, Kersten's results [14] seem more similar to our results.

In the case of soil type 4, which is unsaturated unfrozen [14] with unfrozen water, the estimated values using the present theory seem to be much better than all the models used for the comparison up to $S_r = 0.325$.

Soil types 5 and 6 are saturated soils. Soil type 5 [22] is saturated unfrozen. In this case the estimations using the present theory seem to be the best as compared to the values given by the other theories. The maximum error in this case is 8%. For soil type 6 [22], the percentage of maximum error increases to 8.2%.

Soil types 5 and 6 [22] are basically two-phase water and ice saturated systems. As the present model is quite reliable for two-phase systems (Table 2—maximum error 0–10.9%), the estimations using the present theory for soil types 5 and 6 are rigorous as compared to the models [9, 14, 16].

The present model is based on an integrated theory and is applicable to the dispersions of all kinds of phases. The results for the ETC of two- and three-phase saturated and unsaturated frozen and unfrozen soils using the present model are satisfactory. Due to the accuracy of prediction this model can be used for determining the petroleum content in underground soil. The inaccuracy in prediction lies when the degree of saturation is about 20–30%. In addition the import-

Table 2. Thermal conductivity of air saturated systems (W m^{-1} K^{-1})

System	λ_s	λ_g	ψ_g	λ_{ef} using equation (10)	λ_e using the model of ref. [8]	λ_e experiment	Ref.
Zirconia powder and air	1.998	0.0297	0.42	0.254	0.141	0.229	[19]
Zirconia powder and air	1.998	0.0297	0.36	0.344	0.170	0.363	[19]
Glass beads and air	1.04	0.026	0.46	0.146	0.744	0.150	[20]
Glass beads and air	1.04	0.026	0.476	0.132	0.731	0.130	[20]
Dune sand and air	3.34	0.026	0.4052	0.336	1.67	0.336	[21]
			0.4297	0.299	1.50	0.312	
			0.4394	0.288	1.50	0.302	
			0.460	0.262	1.50	0.274	
Dune sand and air	3.34	0.026	0.480	0.232	1.430	0.239	[11]
			0.485	0.222	1.41	0.220	
Brick sand and air	2.85	0.026	0.490	0.196	0.658	0.195	
Stone concrete and air	2.50	0.026	0.560	0.114	0.104	0.111	

ant aspects which have been ignored in the present derivation are the capillarity and surface tension which could cause some of the deviations between theory and experiment.

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CONDUCTION A TRAVERS DES MATERIAUX GRANULAIRES NON SATURES ET FORMATION DE MILIEUX EFFECTIFS

Résumé—Une théorie pour la conductivité thermique effective (ETC) d'un milieu saturé ou non, contenant trois phases, est présentée en utilisant l'approximation du milieu continu équivalent (ECM). La condition limite pour ECM est tout d'abord dérivée de la technique de dispersion successive. Des petites dispersions sont ensuite considérées dans ECM proposé. La méthode est analytique et elle montre la validité et l'émergence de ECM pour les grandes dispersions. La théorie est applicable pour tous les fluides saturants en incluant les gaz, et elle est utile pour déterminer la présence des produits pétroliers dans le sous-sol. Une comparaison entre les valeurs calculées de ETC et des valeurs expérimentales, pour des fluides saturants qui sont l'air et l'eau, montre un bon accord.

SCHEINBARE STOFFANORDNUNG UND WÄRMELEITUNG IN UNGESÄTTIGTEN PORÖSEN MATERIALIEN

Zusammenfassung—Es wird eine Theorie für die effektive Wärmeleitfähigkeit (ETC) in einem ungesättigten und gesättigten porösen Medium, das drei Phasen enthält, mit der Annäherungsmethode für scheinbar zusammenhängende Stoffe (ECM) vorgestellt. Zuerst wird die Randbedingung für ECM mit einem Verfahren der sukzessiven Einteilung abgeleitet. Die Theorie ist auf alle gesättigten Fluide mit Gasanteilen anwendbar und ist bei der Bestimmung des Gehalts petroleumähnlicher Stoffe im Erdreich hilfreich. Ein Vergleich der mit ETC berechneten Werte mit Experimenten zeigt gute Übereinstimmung mit Luft und Wasser als Hohlraumfluid.

МОДЕЛИ ЭФФЕКТИВНЫХ СРЕД И ТЕПЛОПРОВОДНОСТЬ НЕНАСЫЩЕННЫХ ГРАНУЛИРОВАННЫХ МАТЕРИАЛОВ

Аннотация—С помощью представления ненасыщенных и насыщенных материалов с тремя фазами как эффективных сплошных сред (ЭСС) предложена теория эффективной теплопроводности (ЭТП). Впервые, используя методику последовательной дисперсности, получены граничные условия ЭСС. В предложенной ЭСС возможна малая дисперсность. Данный аналитический метод является эффективным и выявляет ЭСС для больших дисперсностей. Теория применима ко всем насыщающим жидкостям, включая газ, и полезна при определении количества нефтей в пластах. Для случая, когда в качестве насыщающих жидкостей использованы воздух и вода, найдено хорошее соответствие при сравнении расчетных и экспериментальных значений ЭТП.